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Heavy Flavor Theory: Overview*

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Abstract

An introduction to the heavy quark effective theory and its symmetries is given. Some implications of the heavy quark spin and flavor symmetries are discussed. Recent results on fragmentation to quarkonium states are reviewed.

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1. Introduction

In the last few years there has been considerable progress in understanding the physical properties of hadrons containing a single heavy quark. Much of this progress has arisen from the development of the heavy quark effective theory (HQET) and the application of its spin-flavor symmetries.[1,2] The heavy quark symmetries have implications for the physics of hadrons containing a single heavy quark in a kinematic regime where nonperturbative strong interaction physics is important. Here the situation is analogous to the application of the approximate light quark flavor symmetries (e.g., isospin or chiral $SU(2)_L \times SU(2)_R$). In this talk I will review the basic elements of the heavy quark effective theory and give some applications of its symmetries.

Like isospin or $SU(3)$ symmetry the heavy quark symmetries are approximate. Of the six quarks that exist in nature the top, bottom and charm can potentially be treated as heavy. The heavy quark symmetries arise in the limit of QCD where the heavy quark masses m_Q are taken to infinity. In the real world, where the heavy quark masses are finite, corrections to the predictions based on heavy quark symmetry of order Λ_{QCD}/m_Q arise. Here Λ_{QCD} refers to a typical hadronic scale not necessarily the parameter that occurs in the strong coupling constant. The top quark is very heavy, however, it's lifetime is so short that ideas based on heavy quark symmetry have little relevance for it's properties. (It doesn't live long enough to form a hadron.) The charm quark mass is not large enough for us to have confidence that the Λ_{QCD}/m_c corrections are negligible. However, by comparing the predictions of heavy quark symmetry with experiment eventually it will be possible to determine how good the $m_c \to \infty$ limit is. There are already several indications that heavy quark spin symmetry is a useful concept for the charm quark, however, the applicability of the flavor symmetry that relates the properties of hadrons containing a charm quark to those containing a bottom quark has hardly been tested.

In the next section two derivations of the heavy quark effective theory are presented. One derives the HQET by taking a limit of the Feynman rules of QCD

to get the Feynman rules of the effective theory. The second derivation relates the heavy quark fields in the full theory to those of the effective theory. Spectroscopic applications are presented in the third section. This includes a discussion of some very recent work on the application of heavy quark symmetry to fragmentation. The fourth section contains a discussion of the sources of symmetry breaking. Applications to matrix elements relevant for exclusive weak semileptonic decays are discussed in Section 5. This includes a brief presentation of some predictions that arise when chiral $SU(2)_L \times SU(2)_R$ symmetry is combined with heavy quark symmetry. In Section 6 some very recent work on the application of HQET to inclusive semileptonic B decays is discussed. Unfortunately it indicates that a model independent extraction of V_{ub} (the $b \to u$ Cabibbo-Kobayashi-Maskawa matrix element) from the endpoint region of the electron energy spectrum in semileptonic B decays is not possible.

Not all the recent exciting developments in heavy flavor theory are related to the heavy quark effective theory. There have also been recent advances in our understanding of the properties of $\bar{Q}Q$ quarkonium states. For example, over the past year it was realized that fragmentation functions like $D_{c\to\psi}(z)$ are computable in terms of the ψ wave function at the origin and the charm quark mass (for essentially the same reason that ψ decay to three gluons is computable). Fragmentation to $Q\bar{Q}$ states is discussed in the final section.

2. The Heavy Quark Effective Theory

Consider the situation where a heavy quark Q (i.e., $m_Q \gg \Lambda_{QCD}$) is interacting with light degrees of freedom associated with a momentum scale much less than the heavy quark mass. Then it is appropriate to take the limit of QCD where $m_Q \to \infty$ with the heavy quark's four velocity v^μ held fixed.[1,2] This kinematic situation does occur in nature. For example in its rest frame a B^- meson has the b-quark almost at rest at the center of the meson. The size of the meson, however, is determined by nonperturbative strong interactions and is of order $1/\Lambda_{QCD}$. (For example, in a simple string picture it is the tension of the QCD string that goes between the b-quark and the $\bar{u}$-quark that fixes the size of the meson.) Hence by the uncertainty principle

the typical momentum of the light degrees of freedom is of order Λ_{QCD}.

One way of deriving the effective theory that results from this limit is to directly take the limit of the Feynman rules of QCD. To do this we write the heavy quark's four momentum as

$$p_Q = m_Q v + k \ , \tag{1}$$

k is called the residual momentum and is a measure of how much the heavy quark is off-shell. The QCD propagator for the heavy quark is

$$\frac{i(\not{p}_Q + m_Q)}{(p_Q^2 - m_Q^2 + i\varepsilon)} \ . \tag{2}$$

Now substitute eq. (1) into this neglecting the residual momentum k (in comparison with m_Q) wherever possible. In the numerator of eq. (2) the residual momentum can be neglected giving $m_Q(\not{v} + 1)$. However, if the residual momentum is neglected in the denominator we get zero. Thus the leading part that survives in the denominator is the piece that is linear in the residual momentum $2m_Q vk$. Putting our expressions for the numerator and denominator together we get the propagator for the effective heavy quark theory

$$\frac{i(\not{v} + 1)}{2(v \cdot k + i\varepsilon)} \ . \tag{3}$$

Note that it is independent of the heavy quark mass. The vertex for gluon heavy quark interactions is

$$-ig_s T^A \gamma_\mu \ , \tag{4}$$

in QCD. This vertex always appears between propagators in the calculation of any Green function. Because of (3) we can, in the effective theory where $m_Q \to \infty$, replace (4) by

$$-ig_s T^A \frac{(\not{v} + 1)}{2} \gamma_\mu \frac{(\not{v} + 1)}{2} \ . \tag{5}$$

Anticommuting the $\not{v}$ on the right through the γ_μ using $\gamma^\mu(\not{v} + 1) = 2v^\mu - (\not{v} - 1)\gamma^\mu$,

eq. (5) becomes

$$-ig_s T^A v_\mu \ . \tag{6}$$

The propagator in eq. (3) only has a single pole in the complex k^0 plane. Thus heavy quark loops vanish since the contour of the k^0 integral can always be closed in the upper half plane giving zero. There is no heavy quark pair creation in the effective theory.

With the vertex for gluon interactions of the form in eq. (6) factors of the projection operator $(\not{v}+1)/2$ in the heavy quark propagator can be moved to the outside of any Feynman diagram. Then the heavy quark propagator becomes

$$\frac{i}{v\cdot k + i\varepsilon} \ . \tag{7}$$

Eqs. (6) and (7) are the Feynman rules for the HQET. They are independent of the heavy quark mass m_Q. Note also that the gamma matrices have completely disappeared indicating that the heavy quark's spin is conserved.

It is also instructive to understand the relation between heavy quark fields in the HQET and fields in QCD. For QCD the part of the Lagrange density that contains the heavy quark field Q is

$$\mathcal{L} = \bar{Q}(i\not{D} - m_Q)Q \tag{8}$$

where

$$D_\mu = \partial_\mu + ig_s T^A A_\mu^A \ , \tag{9}$$

is the covariant derivative. Writing[2]

$$Q = e^{-im_Q v\cdot x} h_v^{(Q)} \ , \tag{10}$$

where

$$\not{v} h_v^{(Q)} = h_v^{(Q)} \ , \tag{11}$$

The Lagrange density in eq. (8) becomes

$$\mathcal{L} = \bar{h}_v^{(Q)} (m_Q(\not{v} - 1) + i\not{D}) h_v^{(Q)}$$

$$= \bar{h}_v^{(Q)} i\not{D} h_v^{(Q)} \ . \tag{12}$$

Finally using the constraint in eq. (11) to insert factors of $(\not{v} + 1)/2$ on either side of the $\not{D}$ in the last line of eq. (12) the Lagrange density becomes[2]

$$\mathcal{L} = \bar{h}_v^{(Q)} iv \cdot D h_v^{(Q)} \ . \tag{13}$$

The Lagrange density in eq. (13) reproduces the Feynman rules in eqs. (6) and (7) and eqs. (10) and (11) give the relation between the heavy quark field in the effective theory, $h_v^{(Q)}$, and the heavy quark field in the full theory Q.

Strong interactions of a heavy quark depend on its four velocity v_μ but not on its spin or mass. Hence, if there are N_f heavy quarks with the same four velocity, then the effective heavy quark theory has a SU(2N$_f$) spin flavor symmetry. In practice it is the charm and bottom quarks for which this symmetry is relevant. Because $m_c = 1.5 GeV$ is not that large we expect significant Λ_{QCD}/m_c corrections to the predictions of heavy quark symmetry (which become exact in the $m_c \to \infty$ limit).

In the following sections several applications of heavy quark symmetry are given. Corrections to the predictions of heavy quark symmetry are also discussed. Applications of heavy quark symmetry fall into two broad classes, spectroscopic applications and applications to weak decays.

3. Spectroscopy

Because of heavy quark spin symmetry (in the rest frame of the heavy quark) both the spin of the heavy quark $\vec{S}_Q$ and the total angular momentum $\vec{S}$ commute with the Hamiltonian. For the spectroscopy of hadrons containing a single heavy

quark Q the spin of the light degrees of freedom

$$\vec{S}_\ell = \vec{S} - \vec{S}_Q \ , \tag{14}$$

plays an important role. Since both $\vec{S}$ and $\vec{S}_Q$ commute with the Hamiltonian $\vec{S}_\ell$ also commutes with the Hamiltonian. Thus states at rest are labeled not only by their total spin s but also by the spin of the light degrees of freedom s_ℓ.[3] For example the ground state heavy mesons with $Q\bar{q}(q = u$ or $d)$ flavor quantum numbers have $s_\ell = 1/2$ and negative parity. Combining this spin of the light degrees of freedom with the spin of the heavy quark $s_Q = 1/2$ gives meson states $|P^{(Q)}\rangle$ and $|P^{*(Q)}\rangle$ that have total spins $s = 0$ and $s = 1$ respectively. For $Q = c$ these are the D and D^* mesons while for $Q = b$ they are the B and B^* mesons. Writing

$$|P^{(Q)}\rangle = \frac{1}{\sqrt{2}} \left\{ |\uparrow\rangle_Q |\downarrow\rangle_\ell - |\downarrow\rangle_Q |\uparrow\rangle_\ell \right\} \tag{15a}$$

$$|P^{*(Q)}\rangle = \frac{1}{\sqrt{2}} \left\{ |\uparrow\rangle_Q |\downarrow\rangle_\ell + |\downarrow\rangle_Q |\uparrow\rangle_\ell \right\} \tag{15b}$$

it is easy to see that

$$S_Q^3 |P^{(Q)}\rangle = \frac{1}{2} |P^{*(Q)}\rangle \ . \tag{16}$$

In eqs. (15) the first arrow refers to the spin of the heavy quark along the quantization axis ($\hat{z}$) and the second arrow refers to the spin of the light degrees of freedom along the quantization axis. In eqs. (15b) and (16) $|P^{*(Q)}\rangle$ refers to the spin one state with zero spin along the quantization axis. Eq. (16) implies that the $|P^{(Q)}\rangle$ and $|P^{*(Q)}\rangle$ states have the same mass.

In general the spectrum of hadrons containing a single heavy quark Q has (in the $m_Q \to \infty$ limit) for each spin of the light degrees of freedom s_ℓ a degenerate doublet with total spins s_+ and s_- where

$$s_\pm = s_\ell \pm 1/2 \ , \tag{17}$$

(Except for the case $s_\ell = 0$ where the total spin must be 1/2.)

States in a doublet associated with s_ℓ can decay to states in a lower doublet associated with s'_ℓ by emission of light quanta (e.g., $\pi, \pi\pi, \eta, \eta', \rho$, etc.) with total angular momentum L. The decay amplitudes for the possible transitions are related by[3,4]

$$\mathcal{A}(s, s^3 \to s' s'^3 + Lm)$$

$$= R_{s_\ell, s'_\ell, L} (-1)^s \sqrt{2s_\ell + 1}\sqrt{2s' + 1} \begin{Bmatrix} s'_\ell & s_\ell & L \\ s & s' & 1/2 \end{Bmatrix}$$

$$\cdot \langle L, m; s', s'^3 | s, s^3 \rangle \ , \tag{18}$$

in the $m_Q \to \infty$ limit. Here R is a reduced matrix element that depends on s_ℓ, s'_ℓ, L and other possible quantum numbers that label the doublets. It is not surprising that the four transitions $s_\pm = s'_\pm + Lm$ are determined by a single reduced matrix element. Because of heavy quark spin symmetry there is really only one fundamental transition $s_\ell \to s'_\ell + Lm$. The angular distribution of the decay products is determined by

$$\mathcal{A}(\Omega) = \sum_m Y_{Lm}(\Omega) \mathcal{A}(s, s^3 \to s', s'^3 + Lm) \ , \tag{19}$$

for fixed s^3 and s'^3.

For $Q = c$ an excited heavy meson multiplet with $s_\ell = 3/2$ and positive parity has been observed. It is composed of the spin two state $D_2^*(2460)$ and the spin one state $D_1(2420)$. Eq. (18) predicts that the relative partial widths

$$\Gamma(D_2^* \to [\pi D]_{L=2}) : \ \Gamma(D_2^* \to [\pi D^*]_{L=2}) : \ \Gamma(D_1 \to [\pi D^*]_{L=2}) \ , \tag{20}$$

are

$$2/5 \qquad : \qquad 3/5 \qquad : \qquad 1 \tag{21}$$

and that

$$\Gamma(D_1 \to [\pi D^*]_{L=0}) = 0 \ . \tag{22}$$

Eqs. (20), (21) and (22) hold in the $m_c \to \infty$ limit. There is, however, an

important Λ_{QCD}/m_c correction of kinematic origin that must be taken into account. For small $|\vec{p}_\pi|$ we expect the momentum dependence of these decay amplitudes to be of the form $|\vec{p}_\pi|^{2L+1}$. Since the $D^* - D$ mass difference is not very small compared with the $D - D_2^*$ mass difference and the power $2L+1 = 5$ (for $L = 2$) is large it is a bad approximation to treat $|\vec{p}_\pi|$ as the same for the four transitions. Thus, when comparing with experiment, we must multiply eq. (21) by factors of $|\vec{p}_\pi|^5$. These factors are

$$3.3 \quad : \quad 0.88 \quad : \quad 0.57 \tag{23}$$

in units of $10^{-2} GeV^5$. Multiplying (21) by (23) gives predictions for the relative decay rates that include the spin symmetry violation arising from the $D^* - D$ and $D_2^* - D_1$, mass differences. For example,

$$\Gamma(D_2^* \to D\pi)/\Gamma(D_2^* \to D^*\pi) = 2.5 \tag{24}$$

follows from eqs. (21) and (23) and is in good agreement with the experimental value 2.4 ± 0.7. Eq. (22) is also in agreement with observation. If the S-wave $D_1 \to \pi D^*$ amplitude were the "typical" size expected on the basis of our experience with hadronic physics, the D_1 width would be $\gtrsim 100$ MeV. Since the D_1 width is only about 20 MeV the S-wave $D_1 \to \pi D^*$ amplitude is small.[5] However because D-wave decay amplitudes are usually much smaller than S-wave decay amplitudes it is possible that the small order Λ_{QCD}/m_c S-wave $D_1 \to \pi D^*$ amplitude contributes significantly to the D_1 width.

Heavy quark spin symmetry relates fragmentation probabilities for members of a doublet. It implies that the probability, $P^{(H)}_{Q,h_Q \to s,h_s}$, of a heavy quark Q with helicity h_Q (along the fragmentation axis) fragmenting to heavy hadron H in a doublet with spin of the light degrees s_ℓ, total spin s and total helicity (along the fragmentation axis) h_s is[4]

$$P^{(H)}_{Q,h_Q \to s,h_s} = P_{Q \to s_\ell} \sum_{h_\ell} p_{h_\ell} |\langle s_Q, h_Q; s_\ell, h_\ell | s, h_s \rangle|^2 \tag{25}$$

Here $P_{Q \to s_\ell}$ is the probability of the heavy quark fragmenting to a doublet with spin

of the light degrees of freedom s_ℓ. It is independent of the heavy quarks helicity but will depend on other quantum numbers needed to specify the doublet. p_ℓ is the conditional probability that the light degrees of freedom have helicity h_ℓ (given that Q fragments to s_ℓ). The probability interpretation implies that $0 \leq p_{h_\ell} \leq 1$ and

$$\sum_{h_\ell} p_{h_\ell} = 1 \ . \tag{26}$$

Parity invariance of the strong interactions implies that

$$p_{h_\ell} = p_{-h_\ell} \ . \tag{27}$$

Eqs. (26) and (27) restrict the number of independent probabilities p_{h_ℓ} to be equal to $s_\ell - 1/2$ for mesons and s_ℓ for baryons.

Parity invariance of the strong interactions implies that

$$P^{(H)}_{Q,h_Q \to s,h_s} = P^{(H)}_{Q,-h_Q \to s,-h_s} \ . \tag{28}$$

Heavy quark spin symmetry reduces the number of independent fragmentation probabilities. For mesons with spin of the light degrees of freedom s_ℓ the fragmentation probabilities, $P^{(H)}_{Q,h_Q \to s,h_s}$, are expressed in terms the $s_\ell - 1/2$ (s_ℓ for baryons) independent p_{h_ℓ}'s and $P_{Q \to s_\ell}$.

For the D and D^* mesons $s_\ell = 1/2$ and eqs. (26) and (27) give $p_{1/2} = p_{-1/2} = 1/2$. The relative fragmentation probabilities

$$P^{(D)}_{c,1/2 \to 0,0} \quad : \quad P^{(D^*)}_{c,1/2 \to 1,1} \quad : \quad P^{(D^*)}_{c,1/2 \to 1,0} \quad : \quad P^{(D^*)}_{c,1/2 \to 1,-1} \tag{29}$$

are

$$1/4 \qquad : \qquad 1/2 \qquad : \qquad 1/4 \qquad : \qquad 0 \ . \tag{30}$$

Eq. (28) determines the fragmentation probabilities for a helicity $-1/2$ charm quark in terms of those above. The relative fragmentation probabilities for the three D^*

helicities agree with experiment, however, the prediction that the probability for a charm quark to fragment to a D is one third the probability to fragment to a D^* does not. This violation of heavy quark spin symmetry may have its origin in the $D^* - D$ mass difference which suppresses the fragmentation to D^*'s.

Fragmentation probabilities for hadrons containing different heavy quarks Q are equal by heavy quark flavor symmetry. In the $m_Q \to \infty$ limit the shape of a heavy hadrons fragmentation function is, $\delta(1-z)$, since the heavy quark carries all the hadron's momentum (this is for the fragmentation function $D_{Q\to H}(z)$ evaluated at a subtraction point $\mu \approx m_Q$). For finite m_Q the fragmentation functions support is concentrated in a region of z near one of order Λ_{QCD}/m_Q.[6]

4. Λ_{QCD}/m_Q Corrections

The sources of heavy quark symmetry breaking that arise because of the finite value of m_Q are understood. Including effects of order Λ_{QCD}/m_Q the Lagrange density for the heavy quark effective theory is[7]

$$\mathcal{L} = \bar{h}_v^{(Q)} iv\cdot D h_v^{(Q)} + \frac{1}{2m_Q}\bar{h}_v^{(Q)}(iD)^2 h_v^{(Q)}$$

$$-a_2\frac{1}{4m_Q}\bar{h}_v^{(Q)} g_s\sigma^{\mu\nu}G_{\mu\nu}^A T^A h_v^{(Q)} + \mathcal{O}(1/m_Q^2)\ . \tag{31}$$

In the leading logarithmic approximation[8]

$$a_2(\mu) = \left[\frac{\alpha_s(m_Q)}{\alpha_s(\mu)}\right]^{9/(33-2N_f)}\ . \tag{32}$$

The corrections to the $m_Q \to \infty$ limit have a simple physical origin. The second term in eq. (31) is the kinetic energy of the heavy quark. It breaks the heavy quark flavor symmetry but not the spin symmetry. The last term is the energy that arises from the chromomagnetic moment of the heavy quark. The factor $a_2(\mu)$ arises because the operator $\bar{h}_v^{(Q)} g_s\sigma^{\mu\nu}G_{\mu\nu}^A T^A h_v^{(Q)}$ requires renormalization. There is no renormalization point dependence to the heavy quark kinetic energy coefficient

because of reparametrization invariance.[9] This last term breaks both the heavy quark spin and flavor symmetries.

The second and third terms in eq. (31) influence hadronic masses. It is convenient to introduce dimensionless quantities $K_Q(H^{(Q)})$ and $G_Q(H^{(Q)})$ that take this into account. For hadronic states $|H^{(Q)}\rangle$ normalized to unity we define

$$K_Q(H^{(Q)}) = \frac{1}{m_Q^2}\langle H^{(Q)}|\frac{1}{2}\bar{h}_v^{(Q)} D^2 h_v^{(Q)}|H^{(Q)}\rangle \tag{33a}$$

and

$$G_Q(H^{(Q)}) = \frac{a_2}{m_Q^2}\langle H^{(Q)}|\frac{1}{4}\bar{h}_v^{(Q)} g_s \sigma^{\mu\nu} G_{\mu\nu}^A T^A h_v^{(Q)}|H^{(Q)}\rangle \ . \tag{33b}$$

K_Q is proportional to $(\vec{D}h_v)^\dagger(\vec{D}h_v)$ and doesn't (because of reparametrization invariance) have any subtraction in its definition. It follows that K_Q is positive.

The matrix elements in (33) are evaluated in the $m_Q \to \infty$ limit and are independent of the heavy quark masses. It is the coefficients $1/m_Q^2$ and $a_2(\mu)/m_Q^2$ that contain the dependence on m_Q.

The $s_\ell = 1/2$ negative parity ground state meson doublet has spin zero and spin one mesons which we denoted by $P^{(Q)}$ and $P^{*(Q)}$. Up to terms of order $(\Lambda_{QCD}/m_Q)^2$ the masses of these mesons are

$$M(P^{(Q)}) = m_Q + \bar{\Lambda}(P^{(Q)}) + m_Q K_Q(P^{(Q)}) + m_Q G_Q(P^{(Q)}) \tag{34a}$$

$$M(P^{*(Q)}) = m_Q + \bar{\Lambda}(P^{(Q)}) + m_Q K_Q(P^{(Q)}) - \frac{1}{3} m_Q G_Q(P^{(Q)}) \ . \tag{34b}$$

The lowest lying positive parity $s_\ell = 3/2$ excited meson doublet has spin two and spin one members that I denote by $P_2^{*(Q)}$ and $P_1^{(Q)}$ (for $Q = c$ there are the $D_2^*(2460)$

and $D_1(2420)$ mesons). The masses of these hadrons are[10]

$$M(P_1^{(Q)}) = m_Q + \bar{\Lambda}(P_1^{(Q)}) + m_Q K_Q(P_1^{(Q)}) + m_Q G_Q(P_1^{(Q)}) \tag{35a}$$

$$M(P_2^{*(Q)}) = m_Q + \bar{\Lambda}(P_1^{(Q)}) + m_Q K_Q(P_1^{(Q)}) - \frac{3}{5} m_Q G_Q(P_1^{(Q)}) \ . \tag{35b}$$

In eqs. (34) and (35) $\bar{\Lambda}$ is a positive contribution to the heavy meson mass that is independent of m_Q. It comes from the part of the QCD Hamiltonian involving only the light degrees of freedom. A rigorous lower bound on $\bar{\Lambda}(P^{(Q)})$ has recently been derived.[11] In eqs. (34) and (35) we used $G_Q(P^{(Q)}) = -3G_Q(P^{*(Q)})$ and $G_Q(P_1^{(Q)}) = -(5/3)G_Q(P_2^{*(Q)})$. These relations follow from the fact that hadronic matrix elements of $\bar{h}_v^{(Q)} g_s \sigma^{\mu\nu} G_{\mu\nu}^A T^A h_v^{(Q)}$ are proportional to

$$2\vec{S}_\ell \cdot \vec{S}_Q = [\vec{S}^2 - \vec{S}_\ell^2 - \vec{S}_Q^2] = [s(s+1) - s_\ell(s_\ell + 1) - 3/4] \ .$$

This operator causes the splitting between members of doublets

$$m_Q G_Q(P^{(Q)}) = \frac{3}{4}[M(P^{(Q)}) - M(P^{*(Q)})] \tag{36a}$$

$$m_Q G_Q(P_1^{(Q)}) = \frac{5}{8}[M(P_1^{(Q)}) - M(P_2^{*(Q)})] \ . \tag{36b}$$

Comparing eqs. (36) with the measured D, D^*, D_1 and D_2^* masses gives $G_c(D) \simeq -0.06$ and $G_c(D_1) = -0.02$. The fact that these order $(\Lambda_{QCD}/m_c)^2$ quantities are small indicates that the $m_c \to \infty$ limit is a useful approximation. From the known dependence of $G_Q(P^{(Q)})$ on the heavy quark mass it follows that[8]

$$(m_{B^*} - m_B) = \left(\frac{m_c}{m_b}\right) \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{9/25} (m_{D^*} - m_D) \ , \tag{37}$$

which is in agreement with experiment. It is probably a mistake to view this success as very important since a similar formula is known to hold even for light hadrons.

However, the validity of eq. (37) does provide some support for the usefulness of heavy quark flavor symmetry in relating properties of hadrons containing a b-quark to those containing a c-quark. We also have

$$(m_{B_2^*} - m_{B_1}) = \left(\frac{m_c}{m_b}\right)\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{9/25}(m_{D_2^*} - m_{D_1}) \ . \tag{38}$$

We can take a linear combination of eqs. (34) and of eqs. (35) for which G_b cancels out. Define

$$M(P^{(Q)})_{avg} = \frac{M(P^{(Q)}) + 3M(P^{*(Q)})}{4} \tag{39a}$$

$$M(P_1^{(Q)})_{avg} = \frac{3M(P_1^{(Q)}) + 5M(P_2^{*(Q)})}{8} \tag{39b}$$

$$M(P^{(Q)})_{avg} = m_Q + \bar{\Lambda}(P^{(Q)}) + m_Q K_Q(P^{(Q)}) \tag{40}$$

$$M(P_1^{(Q)})_{avg} = m_Q + \bar{\Lambda}(P_1^{(Q)}) + m_Q K_Q(P_1^{(Q)}) \ . \tag{41}$$

These relations (and the known dependence of K_Q on m_Q) imply, for example that,

$$[M(D_1)_{avg} - M(D)_{avg}] - [M(B_1)_{avg} - M(B)_{avg}]$$

$$= m_b\left(\frac{m_b}{m_c} - 1\right)(K_b(B_1) - K_b(B)) \ . \tag{42}$$

Similar mass formulae hold for the baryons. They are particularly simple for the ground state isospin zero baryon which has $s_\ell = 0$. For this Λ_Q baryon $G_Q(\Lambda_Q) = 0$

and

$$M(\Lambda_Q) = m_Q + \bar{\Lambda}(\Lambda_Q) + m_Q K_Q(\Lambda_Q) \ . \tag{43}$$

Combining eqs. (43), (39a) and (34) gives

$$[M(\Lambda_c) - M(D)_{avg}] - [M(\Lambda_b) - m(B)_{avg}]$$

$$= m_b \left(\frac{m_b}{m_c} - 1 \right) (K_b(\Lambda_b) - K_b(B)) \ . \tag{44}$$

The measured values of the $\Lambda_c, \Lambda_b, D, D^*, B$ and B^* masses implies from eq. (44) that the difference between the heavy quark kinetic energies in the Λ_b and B is small.

5. Exclusive Semileptonic Decays

Heavy quark flavor symmetry plus isospin symmetry implies (again heavy meson states are normalized to unity) that

$$\langle L(k,\epsilon)|\bar{u}\gamma_\mu(1-\gamma_5)b|B(v)\rangle$$

$$= \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} \langle L(k,\epsilon)|\bar{d}\gamma_\mu(1-\gamma_5)c|D(v)\rangle \tag{45}$$

where L is a state that doesn't contain a heavy quark (i.e., L = vacuum, pion, rho, etc.). The factor of $[\alpha_s(m_b)/\alpha_s(m_c)]^{-6/25}$ in eq. (45) arises from the relationship between currents in the full theory of QCD and currents in the effective theory[12]

$$\bar{q}\gamma_\mu(1 \pm \gamma_5)Q = \left[\frac{\alpha_s(m_Q)}{\alpha_s(\mu)} \right]^{-6/(33-2N_f)} \bar{q}\gamma_\mu(1 \pm \gamma_5)h_v^{(Q)} \ . \tag{46}$$

Eq. (45) holds for $v \cdot k \ll m_{c,b}$ which insures that momentum transfers associated with the light degrees of freedom are small compared with the heavy quark masses.

The simplest choice for L in eq. (45) is the vacuum. Then (45) gives a relation between decay constants[12]

$$f_B = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} \sqrt{\frac{m_D}{m_B}} f_D \ . \tag{47}$$

There is evidence from 2-D QCD in the large N_c limit,[13] QCD sum rules,[13] the nonrelativistic constituent quark model and (most importantly) Lattice Monte Carlo calculations[15] that the Λ_{QCD}/m_c corrections to eq. (47) are very large.

In principle, eq. (45) together with data on $B \to Le\bar{\nu}_e$ and $D \to L\bar{e}\nu_e$ can be used to determine the $b \to u$ weak mixing angle $|V_{ub}|$. The point is that since the mixing angles are known for the D decay case experimental data on $D \to L\bar{e}\nu_e$ can be used to determine the r.h.s. of eq. (45). There are, however, complications that arise because typically not all the form factors that characterize these matrix elements are measurable. Consider for definitness the care where L is a single pion. The $B \to \pi$ matrix element of the axial current vanishes (because of parity invariance of the strong interactions) and the vector current matrix element is characterized by two Lorentz invariant form factors that can be taken to be functions of $v \cdot k$,

$$\langle \pi(k) | \bar{u}\gamma_\mu b | B(v) \rangle = \frac{1}{\sqrt{2m_B}} [f_+^{(B\to\pi)}(p_B + k)_\mu + f_-^{(B\to\pi)}(p_B - k)_\mu] \ , \tag{48}$$

where $p_B = m_B v$. The l.h.s. of eq. (48) is independent of m_b (for large m_b) and so

$$f_+ + f_- \sim \mathcal{O}(1/\sqrt{m_b}) \ , \tag{49a}$$

$$f_+ - f_- \sim \mathcal{O}(\sqrt{m_b}) \ . \tag{49b}$$

Eq. (48) implies that (for large m_b) $f_+ + f_-$ is much smaller than $f_+ - f_-$ or

equivalently $f_+ \simeq -f_-$. Using eq. (45) for $L = \pi$ gives[16]

$$(f_+ + f_-)^{(B\to\pi)} = \left(\frac{m_D}{m_B}\right)^{1/2} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} (f_+ + f_-)^{(D\to\pi)} \ , \tag{50a}$$

$$(f_+ - f_-)^{(B\to\pi)} = \left(\frac{m_B}{m_D}\right)^{1/2} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} (f_+ - f_-)^{(D\to\pi)} \ . \tag{50b}$$

Neglecting the electron mass the matrix elements for $B \to \pi e \bar{\nu}_e$ and $D \to \pi \bar{e} \nu_e$ depend only on f_+. However, eqs. (50) relate $f_+^{(B\to\pi)}$ to a linear combination of $f_+^{(D\to\pi)}$ and $f_-^{(D\to\pi)}$. Fortunately we can use $f_+ \simeq -f_-$ to get a relation between the physically measurable quantities

$$f_+^{(B\to\pi)} = \left[\frac{m_B}{m_D}\right]^{1/2} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} f_+^{(D\to\pi)} \ . \tag{51}$$

If SU(3) is used instead of isospin then $f_+^{(D\to\pi)}$ on the r.h.s. of eq. (51) can be replaced by $f_+^{(D\to K)}$ which has already been measured.

Naively eq. (51) is valid as long as $v \cdot k \ll m_{c,b}$ (Quark model estimates suggest that it holds even for $v \cdot k \sim m_{c,b}$). However, there are important corrections that arise for very small $v \cdot k \sim m_\pi$. They occur because of pole graphs involving the B^* (for $B \to \pi$) and D^* (for $D \to \pi$) mesons. The form of the $B \to \pi$ matrix element for very small $v \cdot k$ is determined by chiral perturbation theory which gives[17]

$$(f_+ + f_-)^{(B\to\pi)} = -\left(\frac{f_B}{f_\pi}\right) \left[1 - \frac{g v \cdot k}{(v \cdot k + m_{B^*} - m_B)}\right] \ , \tag{52a}$$

$$(f_+ - f_-)^{(B\to\pi)} = \frac{-g f_B m_B}{f_\pi (v \cdot k + m_{B^*} - m_B)} \ . \tag{52b}$$

In eqs. (52) g/f_π is the $B^* B \pi$ coupling which by heavy quark flavor symmetry is the same as the $D^* D \pi$ coupling. It determines the D^* width

$$\Gamma(D^{*+} \to D^0 \pi^+) = \frac{g^2}{6\pi f_\pi^2} |\vec{k}_\pi|^3 \ , \tag{53}$$

where $f_\pi \simeq 135$ MeV. In the nonrelativistic constituent quark model $g = 1$ (a similar estimate for the pion nucleon coupling gives $g_A = 5/3$).

A formula like (52) also holds for $D \to \pi$ (just replace all the B subscripts by D subscripts). Because the $D^* - D$ mass difference is comparable with m_π, for $v \cdot k \sim m_\pi$, it is not a good approximation to neglect $m_{D^*} - m_D$ compared with $v \cdot k$. Hence, eq. (51) (which neglects order Λ_{QCD}/m_c effects like the $D^* - D$ mass difference) doesn't hold in this kinematic regime.

Our discussion of heavy-hadron to light-hadron semileptonic transitions has focussed on those aspects that may be useful for extracting $|V_{ub}|$. However, it is worth recalling that there are interesting applications of the heavy quark spin symmetry to such transitions that don't bear on this issue. For example, charm quark spin symmetry constrains the form factors for $\Lambda_c \to \Lambda \bar{e} \nu_e$.[18] This prediction of charm quark spin symmetry has recently been verified experimentally.

There are very important applications of heavy quark symmetry for heavy-hadron to heavy-hadron semileptonic transitions. The classic example is $B \to D e \bar{\nu}_e$ and $B \to D^* e \bar{\nu}_e$ semileptonic decays. Lorentz and parity invariance of QCD imply that (recall heavy meson states are normalized to unity instead of twice their mass)

$$\langle D(v') | \bar{c} \gamma_\mu b | B(v) \rangle = \frac{1}{2} [\tilde{f}_+ (v + v')_\mu + \tilde{f}_- (v - v')_\mu] \ , \tag{54a}$$

$$\langle D^*(v', \epsilon) | \bar{c} \gamma_\mu \gamma_5 b | B(v) \rangle = \frac{1}{2} [\tilde{f} \epsilon^*_\mu + \tilde{a}_+ (\epsilon^* \cdot v)(v + v')_\mu + \tilde{a}_- (\epsilon^* \cdot v)(v - v')_\mu] \ , \tag{54b}$$

$$\langle D^*(v', \epsilon) | \bar{c} \gamma_\mu b | B(v) \rangle = \frac{1}{2} i \tilde{g} \epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\nu} v'^\lambda v^\sigma \ . \tag{54c}$$

In eqs. (54) the Lorentz invariant form factors $\tilde{f}_\pm, \tilde{f}, \tilde{a_\pm}$ and $\tilde{g}$ are functions of $v \cdot v'$. Heavy quark spin symmetry implies that all of these form factors are expressed in terms of a single universal function of $v \cdot v'$ (the Isgur-Wise function) $\xi(v \cdot v')$. The

relationship is[1]

$$\tilde{f}_+ = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} \xi(v\cdot v') \ , \quad \tilde{f}_- = 0 \tag{55a}$$

$$\tilde{f} = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} (1 + v\cdot v')\xi(v\cdot v') \tag{55b}$$

$$(\tilde{a}_+ - \tilde{a}_-) = -\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} \xi(v\cdot v') \ , \quad (\tilde{a}_+ + \tilde{a}_-) = 0 \ , \tag{55c}$$

$$\tilde{g} = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} \xi(v\cdot v') \ . \tag{55d}$$

Furthermore the normalization of ξ at the zero recoil point, $v \cdot v' = 1$, is determined by heavy quark flavor symmetry to be[1,19]

$$\xi(1) = 1 \ . \tag{56}$$

The perturbative corrections to eqs. (55) of order $\alpha_s(m_b)$ and $\alpha_s(m_c)$ are calculable and don't cause any loss of predictive power.[20,21] At order $\Lambda_{QCD}/m_{c,b}$ new universal functions enter in the form factors and the predictive power of heavy quark symmetry is greatly diminished. However, at zero recoil, $v \cdot v' = 1$, it has been shown that there are no $\Lambda_{QCD}/m_{c,b}$ corrections.[22] This remarkable result is usually called Luke's theorem and it means that $B \to D^* e\bar{\nu}_e$ decay may eventually provide a very accurate determination of $|V_{cb}|$. Chiral perturbation theory has been used to analyze the order $(\Lambda_{QCD}/m_c)^p, p = 2, 3, ...$ corrections at zero recoil. The corrections, of this order, that have a nonanalytic dependence on the pion mass of the form $\ell n m_\pi$ for $p = 2$ and $(1/m_\pi)^{p-2}$ for $p = 3, 4, ...$ are calculable. They are about a few percent in magnitude.[23] The divergence that occurs as $m_\pi \to 0$ makes all these $(\Lambda_{QCD}/m_c)^p, p = 2, 3...$ corrections of comparable importance.

Perhaps the most elegant application of heavy quark symmetry is to the decay $\Lambda_b \to \Lambda_c e\bar{\nu}_e$. In this situation, the physics is particularly simple since the $\Lambda_{b,c}$ have

$s_\ell = 0$. In the $m_{b,c} \to \infty$ limit all the form factors are again expressible in terms of a single universal function. Also, perturbative order $\alpha_s(m_b)$ and $\alpha_s(m_c)$ corrections are calculable and don't cause any loss of predictive power. However, in this case even the $\Lambda_{QCD}/m_{c,b}$ nonperturbative predictions cause little loss of predictive power.[24] They are calculable in terms of the single quantity $\bar{\Lambda}(\Lambda_{b,c})$ introduced in Section 4. Unlike the meson case no new unknown functions of $v \cdot v'$ arise at order $\Lambda_{QCD}/m_{c,b}$.

6. Inclusive Semileptonic Decays

The inclusive lepton spectrum from semileptonic B decays has undergone intensive experimental and theoretical study. Recently there has been considerable progress in understanding inclusive semileptonic decay. Inclusive semileptonic B-decay can be treated in a fashion similar to deep inelastic scattering. Using a two step process that consists first of an operator product expansion and then a transition to the heavy quark effective theory it can be shown that $d\Gamma/dq^2dE_e$ ($q^2 = (p_e - p_{\bar{\nu}_e})^2$), when suitably averaged over E_e, is calculable.[25] The leading order result is

$$\frac{d\Gamma^{(0)}}{d\hat{q}^2 dy} = \sum_j \frac{|V_{jb}|^2 G_F^2 m_b^5}{192\pi^3} \theta(1 + \hat{q}^2 - \rho - \frac{\hat{q}^2}{y} - y)$$

$$\cdot\{12(y - \hat{q}^2)(1 + \hat{q}^2 - \rho - y)\} \ , \tag{57}$$

where

$$\hat{q}^2 = \frac{q^2}{m_b^2} \ , \quad \rho = \frac{m_j^2}{m_b^2} \ , \quad y = \frac{2E_e}{m_b} \ , \tag{58}$$

and $j = u$ or c. This agrees with free b-quark decay.

The b-quark and c-quark masses that appear in eq. (57) have a precise meaning. These masses are the same heavy quark masses as appear in eq. (10) which describes the transition from QCD to the heavy quark effective theory (i.e., pole masses in the heavy quark propagators).

The full power of this method for treating inclusive semileptonic decay becomes apparent when corrections to the leading result are considered. These corrections are of two types, perturbative $\alpha_s(m_b)$ corrections and nonperturbative corrections suppressed by powers of Λ_{QCD}/m_b. In fact there are no nonperturbative corrections of order Λ_{QCD}/m_b. They first arise at order $(\Lambda_{QCD}/m_b)^2$.

The nonperturbative corrections to eq. (57) of order $(\Lambda_{QCD}/m_b)^2$ are proportional to the quantities $G_b(B)$ and $K_b(B)$ (which were defined in Section 4) and have recently been calculated.[26,27,28] They add the following terms to the differential decay rate[27]

$$\frac{d^2\Gamma^{(2)}}{dyd\hat{q}^2} = \sum_j \frac{|V_{jb}|^2 G_F^2 m_b^5}{192\pi^3}\Bigg\{\theta(1+\hat{q}^2-\rho-\frac{\hat{q}^2}{y}-y)$$

$$\cdot[12E_b(B)(2\hat{q}^4-2\hat{q}^2\rho+y-2\hat{q}^2y+\rho y)+8K_b(B)(2\hat{q}^2-\hat{q}^4+\hat{q}^2\rho-3y)$$

$$+8G_b(B)(-\hat{q}^2+2\hat{q}^4-2\hat{q}^2\rho-2y-2\hat{q}^2y+\rho y)]$$

$$+\delta(1+\hat{q}^2-\rho-\frac{\hat{q}^2}{y}-y)\frac{1}{y^2}[12E_b(B)\hat{q}^2(y-\hat{q}^2)(-\hat{q}^2+2y-y^2)$$

$$+4K_b(B)(-\hat{q}^6+9\hat{q}^4y-6\hat{q}^2y^2-2\hat{q}^4y^2-\hat{q}^2y^4+y^5)$$

$$+8\hat{q}^2G_b(B)(y-\hat{q}^2)(-\hat{q}^2+y+y^2)]$$

$$+\delta'(1+\hat{q}^2-\rho-\frac{\hat{q}^2}{y}-y)K_b(B)\frac{4\hat{q}^2}{y^3}(y^2-\hat{q}^2)^2(y-\hat{q}^2)\Bigg\}\ . \tag{59}$$

In eq. (59) $E_b(B) = K_b(B) + G_b(B)$. The terms in eq. (59) proportional to $E_b(B)$ and $K_b(B)$ have a simple physical interpretation. They can be thought of as arising from a shift in the b-quarks mass and four velocity due to the bound state.

The corrections in eq. (59) are singular along the boundary of the Dalitz plot, $1+\hat{q}^2-\rho-\hat{q}^2/y-y=0$ because of the delta function and derivative of a delta function. This singular behavior is an indication that near the boundary of the Dalitz

plot the theoretical prediction must be smeared over a range of electron energies to be physically meaningful. (Perturbative corrections are also singular here and must also be smeared.) The range of electron energies must be large enough that $d^2\Gamma^{(2)}/dyd\hat{q}^2$ (when smeared) can be treated as a small correction to $d^2\Gamma^{(0)}/dyd\hat{q}^2$. This comparison indicates that near the boundary of the Dalitz plot the electron energy must be smeared over a range of electron energies $\Delta E_e \gtrsim$ 500 MeV. The endpoint region of the electron spectrum, $m_B/2 > E_e > (m_B^2 - m_D^2)/2m_B$, is very important since $b \to c$ transitions cannot contribute there. A theoretical prediction for the normalization of the electron spectrum in this region would allow the extraction of $|V_{ub}|$ from experimental data on the endpoint region electron spectrum. Unfortunately the smearing makes a theoretical prediction (from QCD) of this normalization impossible. Extractions of $|V_{ub}|$ with this method are model dependent. Fortunately as discussed in Section 5 there is hope that $|V_{ub}|$ can be determined from exclusive weak decays. However, much further work is needed on estimating the size of the $\Lambda_{QCD}/m_{c,b}$ corrections in order to assess the viability of this method.[29] The method for calculating nonperturbative corrections to semileptonic decay rates outlined above has also be applied to the inclusive rare decays $B \to X_s e^+ e^-$ and $B \to X_s \gamma$.[30]

7. Fragmentation to Quarkonium

Quarkonium production in high energy processes is dominated by fragmentation of heavy quarks and gluons. For example, in Z^0 decay the short distance process $Z^0 \to \psi gg$ is suppressed relative to the fragmentation process $Z^0 \to \psi c\bar{c}$ by a factor of order m_c^2/m_Z^2. Recently it has been realized that the process in dependent fragmentation functions for quarkonium production in high energy experiments are computable.[31,32]

For definitness consider the fragmentation of charm quarks to ψ's. The fragmentation function to transversely aligned ψ's is[33]

$$D^T_{c\to\psi}(z) = \frac{16}{81 m_c^2}\alpha_s(2m_c)^2 f_\psi^2 \frac{2}{3}\frac{z(1-z)^2}{(2-z)^6}$$

$$\cdot\{16 - 32z + 76z^2 - 36z^3 + 6z^4\} \ , \tag{60}$$

and the total fragmentation function to ψ's (i.e., sum of longitudinal and transverse polarizations) is[30]

$$D_{c\to\psi}^{T+L}(z) = \frac{16}{81m_c^2}\alpha_s(2m_c)^2 f_\psi^2 \frac{z(1-z)^2}{(2-z)^6}$$

$$\cdot\{16 - 32z + 72z^2 - 32z^3 + 5z^4\} \ . \tag{61}$$

In eqs. (60) and (61) the fragmentation functions are evaluated at a subtraction point $\mu \approx 2m_c$. They can be evolved to higher energies using the Altarelli Parisi equations. This evolution will induce a fragmentation function $D_{g\to\psi}(z)$. (At $\mu = 2m_c$ $D_{g\to\psi}(z)$ is of order $\alpha_s(2m_c)^3$.) In eqs. (60) and (61) f_ψ is the ψ decay constant, defined by

$$\langle 0|\bar{c}\gamma_\mu c|\psi(p,\epsilon)\rangle = f_\psi m_\psi \epsilon_\mu \ . \tag{62}$$

The leptonic width of the ψ determines that $f_\psi \simeq 410$ MeV. The total fragmentation probability $P_{c\to\psi} = \int dz D_{c\to\psi}^{T+L}(z)$ is subtraction point independent and eq. (61) gives $P_{c\to\psi} \simeq 2\times 10^{-4}$.

If the expression in the brace brackets of eqs. (60) and (61) were identical the ψ's produced by fragmentation are unaligned. Comparison of eqs. (60) and (61) indicates a very slight preference for transversely aligned psi's. Let ζ be the ratio of transverse to total fragmentation probabilities

$$\zeta = \frac{\int_0^1 dz D_{c\to\psi}^{T}(z)}{\int_0^1 dz D_{c\to\psi}^{T+L}(z)} \ . \tag{63}$$

This ratio is μ independent and using eqs. (60) and (61) we find $\zeta = 0.69$ to be compared with $\zeta = 2/3$ for the production of unaligned ψ's. The ratio ζ is measurable through the angular distribution of the leptons in the decay $\psi \to \ell^+\ell^-$. Defining θ to be the angle between the alignment axis and the lepton momentum the angular

distribution $d\Gamma/d\cos\theta$ in the ψ rest frame has the form

$$\frac{d\Gamma(\psi \to \ell^+\ell^-)}{d\cos\theta} \;\propto\; (1 + \alpha\cos^2\theta) \tag{64}$$

where

$$\alpha = \frac{3\zeta - 2}{2 - \zeta} \;, \tag{65}$$

$\zeta = 0.69$ corresponds to the small asymmetry $\alpha = 0.053$. There are of course other sources of ψ's that must be taken into account at this level. For example fragmentation to P-wave quarkonium that subsequently decays to a ψ can disturb this prediction for α. (ψ's can also arise from production of radially excited states which subsequently decay to $\psi\pi\pi$. However, this production mechanism does not correct the value of ζ).

It is interesting to compare the alignment of ψ's produced by fragmentation with the alignment of ψ's produced by nonleptonic B-decay. Assuming the four-quark amplitude factorizes a straightforward calculation of $b \to \psi s \to \ell^+\ell^- s$ gives

$$\alpha = -\frac{m_b^2 - m_\psi^2}{m_b^2 + m_\psi^2} \approx -0.46 \;, \tag{66}$$

a value very different from ψ's produced by fragmentation. Applications of the ideas reviewed in this section to fragmentation to B_c mesons[31,32] and baryons containing two heavy quarks[34] have also been considered in the recent literature.

8. References

1. N. Isgur and M.B. Wise, Phys. Lett., **B232** 113 (1989); Phys. Lett., **B237** 527 (1990).

2. E. Eichten and B. Hill, Phys. Lett., **B234** 511 (1990); H. Georgi, Phys. Lett., **B240** 447 (1990).

3. N. Isgur and M.B. Wise, Phys. Rev. Lett., **66** 1130 (1991).

4. A.F. Falk and M.E. Peskin, SLAC-PUB-6311 (1993) unpublished.
5. M. Lu, M.B. Wise and N. Isgur, Phys. Rev., **D45** 1553 (1992).
6. R.L. Jaffe and L. Randall, CTP#2189 (1993) unpublished.
7. A.F. Falk, B. Grinstein and M.E. Luke, Nucl. Phys., **B357** 185 (1991); E. Eichten and B. Hill, Phys. Lett., **B243** 427 (1990).
8. G.P. Lepage and B.A. Thacker, Nucl. Phys. B (Proc. Suppl.), **4** 199 (1988).
9. M. Luke and A.V. Manohar, Phys. Lett., **B286** 348 (1992); M.J. Dugan, M. Golden and B. Grinstein, Phys. Lett., **B282** 142 (1992).
10. U. Aglietti, Phys. Lett., **B281** 341 (1992).
11. Z. Guralnik and A. Manohar, Phys. Lett., **B302** 103 (1993).
12. M.B. Voloshin and M.A. Shifman, Sov. J. Nucl. Phys., **45** 292 (1987); H.D. Politzer and M.B. Wise, Phys. Lett., **B206** 681 (1988).
13. M. Burkhardt, Phys. Rev., **D46** 1924 (1992); Phys. Rev. D **46** 2751 (1992); M. Burkhardt and E.S. Swanson, Phys. Rev., **D46** 5083 (1992); B. Grinstein and P.F. Mende, Phys. Rev. Lett., **69** 1018 (1992).
14. M. Neubert, Phys. Rev., **D45** 2451 (1992).
15. See talk by P. Mackenzie and references therein.
16. N. Isgur and M.B. Wise, Phys. Rev., **D42** 2388 (1990).
17. M.B. Wise, Phys. Rev., **D45** 2188 (1992); G. Burdman and J. Donoghue, Phys. Lett., **B208** 287 (1992); T.M. Yan, H.Y. Cheng, C.Y. Cheng, G.L. Lin, Y.C. Lin and H.L. Yu, Phys. Rev., **D46** 1148 (1992).
18. T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys., **B355** 38 (1991); F. Hussain, J.H. Korner, M. Kramer and G. Thompson, Z. Phys., **C51** 321 (1991).
19. S. Nussinov and W. Wetzel, Phys. Rev., **D36** 130 (1987); M.A. Shifman and M.B. Voloshin, Sov. J. Nucl. Phys., **47** 511 (1988).

20. A.F. Falk, H. Georgi, B. Grinstein and M. Wise, Nucl. Phys., **B343** 1 (1990); A. Falk and B. Grinstein, Phys. Lett., **B247** 406 (1990); Phys. Lett., **B249** 314 (1990).

21. M. Neubert, Phys. Rev., **D46** 2212 (1992).

22. M.E. Luke, Phys. Lett., **B252** 447 (1990).

23. L. Randall and M.B. Wise, Phys. Lett., **B303** 139 (1993).

24. H. Georgi, B. Grinstein and M.B. Wise, Phys. Lett., **B252** 456 (1990).

25. J. Chay, H. Georgi and B. Grinstein, Phys. Lett., **B247** 399 (1990).

26. I.I. Bigi, M. Shifman, N.G. Uraltsev and A.I. Vainshtein, Phys. Rev. Lett., **71** 496 (1993).

27. B. Blok, L. Koyrakh, M. Shifman and A.I. Vainshtein, NSF-ITP-93-68, (1993) unpublished; A.V. Manohar and M.B. Wise, CALT-68-1883 (1993) unpublished.

28. T. Mannel, IKDA-93/26 (1993) unpublished.

29. C.O. Dib and F. Vera, Phys. Rev., **D47** 3938 (1993).

30. A.F. Falk, M. Luke and M. Savage, SLAC-PUB-6317 (1993) unpublished.

31. E. Braaten, K. Cheung and T.C. Yuan, NUHEP-TH-93-2 (1993) unpublished; E. Braaten and T.C. Yuan, NUHEP-92-23 (1993) unpublished; E. Braaten, K. Cheung and T.C. Yuan, NUHEP-TH-93-6 (1993) unpublished.

32. Y.Q. Chen and Y.P. Kuang, Phys. Rev., **D46** 1165 (1992); Y.Q. Chen, PhD Thesis, The Institute of Theoretical Physics, Academia Sinica, P.R. of China (1992).

33. A.F. Falk, M. Luke, M. Savage and M.B. Wise, Phys. Lett., **B312** 486 (1993).

34. A.F. Falk, M. Luke, M. Savage and M.B. Wise, CALT-68-1868 (1993) unpublished.